Button Matrix: how Tangible Interfaces can Structure Physical Experiences for Learning

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ABSTRACT
Physical experiences are frequently used to represent mathematics to children. However, students sometimes fail to transfer performance to symbolic representations of problems. In this paper, we suggest that tangible interfaces can promote transfer by structuring physical experiences. We realize our concept in a system, Button Matrix, that uses coupled tactile, vibration and visual feedback to a) *highlight* features of a physical experience that represents arithmetic concepts and b) cue *reflection* on the links between the physical experience and mathematical symbols.

ACM Classification Keywords
H.5.2. [Information Interfaces and Presentation]: User Interfaces-- *input devices and strategies.*

INTRODUCTION
Tangible user interfaces (TUIs) and physical materials (e.g. Diennes Blocks) are frequently used to teach children abstract concepts, as in science and mathematics [7,8]. These media are popular because children often demonstrate knowledge through physical interaction that they cannot in words or in pictures [6,10], and may solve problems better when physically "working them out" [6,7]. However, in many situations, children do not transfer performance with physical to symbolic representations of problems [8]. What is needed are features that promote transfer. Two ways to promote transfer may be: highlighting conceptually relevant physical features [7], and cueing reflection on the links between physical and symbolic representations [10]. Here, we describe how tangible interfaces might accomplish these goals.

THEORETICAL BACKGROUND
Mathematical concepts can be represented in various ways [7], including symbols, graphs, and objects. Generally, students find concrete representations, such as pictures and objects, easier to understand than symbolic representations. [7,8]. However, mathematical aptitude requires fluency with symbolic representations [9]; training students to translate between different representations is a goal of mathematics education [9,4].

Translating between representations presupposes knowledge of the concepts' essential, or 'representation-independent' features [10]. Because concepts must be introduced via a representation, a question in mathematics education is which representation to use? The ideal representation allows students to understand mathematics using knowledge they already have, without downplaying the concepts' essential features, or introducing inessential ones [7,5].

Nunez and Lakoff propose that because mathematics derives from physical experience, teachers should introduce mathematics via physical objects or gesture [6]. This allows students to understand mathematical concepts, such as division, as physical actions, such as partitioning sets of objects. The rationale is that because physical constraints on this action reflect mathematical rules, (one cannot evenly partition an odd set, or divide an odd integer), students can apply such "physical knowledge" to other representations.

Applications of Lakoff's view have met variable success. When learning through physical experience, students are more engaged [7,8], and demonstrate problem-solving strategies they do not in writing or speech [5]; however, students rarely transfer performance to symbolic representations [8].

Failure to transfer knowledge from one representation to another could suggest deficiencies in the representation itself, (it misrepresents some, or cannot represent all, conceptual features), or in learners' comprehension of its relations to the concept. In the latter case, our goal is changing how students experience the representation, versus changing the representational mode.

There is evidence supporting this latter alternative. Goldin-Meadow investigated which factors predicted transfer from gestural to symbolic representations of balance problems; the gesture was intended to highlight a mathematically viable strategy [5]. As a group, the gesturers' pre-post test

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performance improved more than non-gesturers, and they seemed to use the strategy the gesture represented. In support of Lakoff's view, this suggests that physical experiences can develop transferable knowledge. However, not all gesturers improved; those who did could verbalize the strategy the gesture represented. Because *telling* students the strategy did not improve performance, Goldin-Meadow hypothesized that gesture only benefited students inclined to experience the gesture mathematically, i.e., they attended to mathematically relevant features, and interpreted them as mathematical concepts [5].

The question then becomes how to encourage mathematical ways of experiencing physical representations? Goldin-Meadows' work suggests that verbally conveying the relations between representation and concept is insufficient. By Nunez and Lakoff's view, "mathematical ways" of experiencing would also develop through physical experience. Entertaining this perspective, we urge designers to create physical experiences that *encourage* mathematical ways of experiencing them.

Recent work suggests two strategies: a) *highlight* mathematically relevant physical features and b) *cue reflection* on their connections with symbolic representations [7,9]. Both promote transfer in more students. For example, Walkington et al found that "full-bodied" gestures, which highlighted mathematical constraints by mapping them to salient biomechanical constraints, produced transfer in more students than "hand-only" gestures, which did not [13]. However, transfer still depended on students' apprehending mathematically relevant gestural features, such as the "length" of an imaginary hypotenuse [13]. More recently, Gerofsky mapped "full-bodied" gestures to functions, and highlighted relevant gestural features, e.g, extrema and roots, acoustically [4]. She found transfer in all students; however, she measured transfer to graphic representations alone. It thus remains unclear how to help all students apprehend mathematically relevant features, and to relate them to *symbolic* representations. In the following sections, we describe how programmable tangible interfaces could accomplish both goals.

The first goal is highlighting mathematically relevant physical features. "Dually coding" such features in stimuli of another modality, one that is salient and related to the physical action, could help students apprehend them [3]. Programmable interfaces, which 'know' the relevant mathematical features, could control these additional stimuli. Consider the representation "division is partitioning physical sets", in which case "indivisibility" is subset inequality. A programmable interface, which "knows" the set size and current divisor, could require more force to partition indivisible than divisible sets. Students may be more perceptive to the core relation between set size, subset equality and number of subsets when the requisite force to partition correlates it.

Although many modalities could dually-code the relationship between physical action and mathematical concept, work on human motor learning suggests that tangible (e.g., texture, resistance), as opposed to non-tangible (vision and sound) feedback, is ideal: non-tangible feedback may distract learners from their physical experiences [2] and prevent them from learning the concepts the experience represents. Tangible stimuli may also have an advantage because they can directly constrain physical actions, e.g., the increasing resistance of an exercise band directly constrains the ease with which I can stretch it apart. Non-tangible stimuli, by contrast, constrain action indirectly (e.g. "red light" makes us stop because of an additional rule). Stimuli that directly constrain physical action may better reflect how mathematical properties (e.g., divisibility of m by n) constrain mathematical actions (can I evenly partition m objects into n subsets?) [12].

While strictly tangible feedback may meet our first design goal (highlight the action's mathematically relevant features), our second, promote reflection on the connections between different mathematical representations, requires visual-symbolic representations, i.e., of expressions like "2 x 3 =6". Ullmer and Ishii [10] suggest that 'coupling' tangible and visual representations promotes their integration; in most investigations of physical mathematical experience, however, students are exposed to symbolic representations *after* the experience [4,12,7]. Work on multimodal perception may support Ullmer's view: stimuli are integrated better when presented simultaneously, presumably because the common time-course makes them seem related [1], and because students' memories of the stimuli experience less decay [1].

We therefore encourage designers to use coupled tangible and visual (symbolic) materials to highlight mathematically relevant features of physical experience, and promote integration between physical and symbolic representations. In the remaining section, we show how designers might realize both ideas via describing our initial proof of concept, "Button Matrix": a tangible system for learning arithmetic.

DESIGN RATIONALE AND IMPLICATION:

"Button Matrix" is a tangible system that uses coupled physical (texture and vibration) and visual feedback to highlight features of an action (pressing sequences of buttons) that represent basic arithmetic (addition, subtraction and multiplication). Basic arithmetic seemed a fitting domain because it has been targeted by various other tangible systems [7,8] and because students' core problems (appreciating commutativity and the recursive definition of integers [9]) are well-defined.

There are general criteria for a physical experience that represents mathematics and the system that supports it. The 'physical experience' must map unintuitive mathematical to intuitive physical procedures [10]. The system must be easy to learn and use. It must provide interpretable feedback, and allow students to offload 'irrelevant' cognitive load (e.g., keeping track of where they are in the problem) [11]. Every salient feature should be mathematically relevant, and no feature should convey *erroneous* ideas. A successful system must withstand “rough” button presses, and conceal electronics. It should be small enough to sit on students' desks, but large enough to hold students' attention. We advocate additional criteria: a successful system highlights physical features that represent mathematical concepts and promotes on-line reflection on their relation to symbolic representations.

DESIGN SOLUTION

Button Matrix maps arithmetic to action via Nunez and Lakoff's metaphor of “journeying” along a number line [6]. It is intended for grades 1-2. Students duplicate on-screen integer steps in arithmetic problems (e.g. 2+5) by pressing buttons in a continuous sequence along a 3 by 3 matrix; this captures a recursive definition of integers (1 or 1+ an integer) as decomposable sets, which extends grade 1-2 students' knowledge of counting and patterns [9,10], and may be useful in later grades, when students 'balance equations' by identifying integer operands' missing components (e.g. 7+3=5+X) [5].

Button Matrix maps an unintuitive idea (different symbolic expressions, such as 2x3, 3+3, 7-1, are mathematically equivalent) to an intuitive physical fact (moving up 4, then 2, brings me to the same place as 2, then 4, or 3, then 3). Here, we describe how we used coupled tangible and visual feedback to highlight this physical feature (the equivalence of different journeys) and its relation to the mathematical analogue (various procedures can achieve equivalent results).

TECHNICAL FEATURES

The system is an Arduino microcontrolled button matrix that communicates with laptop-run software. The software displays a screen that the microcontroller updates; the software sends each problem to the microcontroller. Button Matrix sits on students' desks; to promote engagement, it covers the desk. A foamcore box hides the electronics. The caps and buttons are fused with modelling clay, and reinforced with wood glue.

INTERFACE FEATURES AND FUNCTIONALITY

The software allows teachers to specify sequences of addition, subtraction and multiplication problems with operands between 0 and 16; the program displays the problems in sequence.

Button Matrix represents problems in steps. Each step is an integer. Addition and subtraction problems have two steps (one for each operand); multiplication problems (mxn) have n steps, each of which is m.

Students complete steps by pressing a number of buttons equal to the integer. An integer begins one button past where the previous terminated. At the end of the problem, students press a button that represents the result. Each press represents a one unit change in the accumulating result; buttons represent different integers. Adding is moving forward; subtraction moving backward.

The screen represents the student's progress in mathematical symbols. It has 3 components: the Stepper, which displays the current step, surrounded by a rectangle, and all steps that preceded it; the accumulator, which displays the current sum that the students' journey represents, and the solution, which appears when students complete the problem, and displays the problem expression, along with the equals sign and the result (e.g., 2x5=10).

The form of the expressions reinforced the equivalence of different arithmetic procedures, and the relationship between these expressions and the physical experience. For example, the Stepper conveyed a key idea- multiplication is iterated addition- by representing the steps of a multiplication problem (nxm) as $n_0+n_1+...n_m$; students only see it as nxm upon completing the problem, when the solution (nxm=nm) appears. This was intended to provide grade 1-2 students, who do not know multiplication or the symbol “x”, a grounding of multiplication as “iterated addition” [9]; it also related multiplication to the physical procedure of executing an additive journey some number of times. For results >9, the accumulator provided an additional representation: the current sum in base 9 (e.g., 11=9x1+2). The base 9 representation reflects the physical action that produces results >9. Because the matrix has 9 buttons, every 9 buttons, students return to the first button; conveying these longer “journeys” as units of 9, or “times around the matrix”, reinforces the connection between multiplication and addition, and between the physical and arithmetic procedures. The on-screen display and physical matrix were coupled. All screen components updated immediately after the physical interface issued tactile feedback; this highlighted and promoted reflection on the links between all symbolic and physical representations of the students' progress through the problem.

The matrix supplied two forms of tactile feedback. The first attributed a pager motor, which signaled correct and incorrect button presses. Correct presses elicited a 50ms pulse; incorrect presses elicited a continuous buzz that ceased when the user pressed the correct button. The second attributed the button's textures. Each button had a unique texture. Because our matrix was 3 by 3, we represented integers texturally in “base 3 notation”. Each button had 1 to 3 small and 0 to 2 large beads. The large beads represented “one 3”; thus, only the second and third column had large beads (1 and 2 respectively). Like the accumulator's representation of results >9 as “times round a 9x9 matrix”, the base 3 notation highlighted how journey segments <=9 could be represented as “times up a column”.

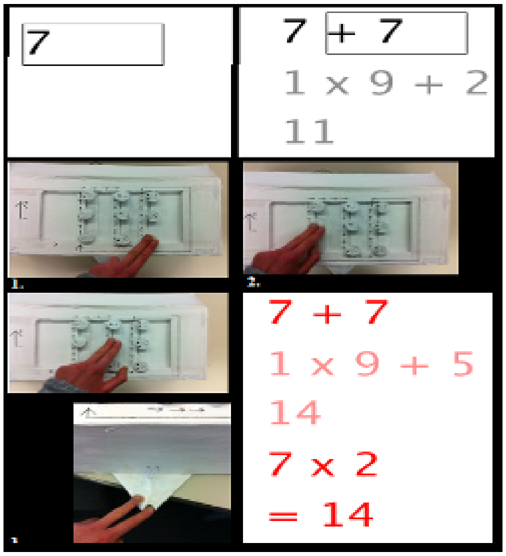

Figure 1: stepping through the problem "7x2=14".

We spaced the beads such that, using two fingers to depress the button, students felt the small and large beads on separate fingers. The short pulse focused students' attention on the feel of the current button, while the sustained pulse washed it out.

The distinctly textured buttons highlighted the physical feature, that various journeys achieve the 'same destination', that represented the arithmetic concept, that various operations achieve the same result. Because of the distinctly textured buttons, mathematically equivalent procedures *feel* the same. Different procedures with equal results (6-2, 2+2), end on the same button; that button feels unique. Because subtraction "goes backward", while addition moves forward, subtraction "feels like" reversing addition. Forcing students to wait until they feel the short pulse prevents them from "rushing" through the journey and promotes attention to the mathematically relevant textures.

USE SCENARIO

Button Matrix is an in-class supplement for grade 1-2 students. We envision students alternating between Button Matrix and paper and pencil to solve arithmetic drill problems. Instructors should specify sets of problems that emphasize equivalence (e.g., 2x3=3+3=4+2=2+4) and are appropriate to the child's skill level. While the instructor should explain and provide examples of how to use Button Matrix, students should be able to complete problems alone.

CONCLUSION

We have articulated a rationale for representing mathematics in physical experiences, and the need for interfaces that promote transfer from physical experiences to symbolic representations. Button Matrix is one realization of our core idea, that by highlighting mathematically relevant physical features, and cueing reflection on their connection to mathematical symbols, tangible interfaces can promote learning through physical experience. Our next step is using Button Matrix to test our ideas. In the meantime, we hope that this conceptual piece inspires designers to consider new requirements for systems supporting embodied learning experiences, and new ways of mapping tangible to conceptual features.

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